Contents lists available at Science-Gate

International Journal of Advanced and Applied Sciences

Journal homepage: http://www.science-gate.com/IJAAS.html

BER performance of homodyne free space optical (FSO) systems over atmospheric turbulence channels

Wansu Lim, Myung-Sik Kim, and Il-Soo Jeon, Hae Chung, Young-Joon Song *

Kumoh National Institute of Technology, Gumi, South Korea

ARTICLE INFO

Article history:
Received 27 December 2015
Received in revised form
25 January 2016
Accepted 26 January 2016

Keywords:
Free space optical (FSO) systems
Atmospheric turbulence channel
Gamma-Gamma distribution
Homodyne systems

ABSTRACT

In this paper, we derived the average bit error rate (BER) in closed-form expression of free space optical (FSO) homodyne systems over atmospheric turbulence channels with Gamma-Gamma distribution. For derivation of the closed-form expression we used special integral and transformation of the Meijer G function. Additionally, we numerically analyzed the average BER behavior according to the average SNR and different turbulence strengths considering several modulation methods including phase shift keying (PSK), amplitude shift keying (ASK), and frequency shift keying (FSK).

1. Introduction

Free space optical (FSO) communication systems (Lim, 2015) provide an attractive and cost effective link for high data rate wireless transmission using highly directive point-to-point links in terrestrial last mile applications between transmitters and receivers along a line of sight (LOS). Moreover, due to the quick installation of transceivers, FSO systems can be deployed in densely populated urban areas or in the battle field. Since optical signals are transmitted through free space medium, FSO systems can be adversely affected by atmospheric conditions. Among various atmospheric conditions, scintillation has the most significant effect or degrades signal strength and link performance. As scintillation results in varying refractive indexes of the signal, the intensity and phase of the optical signal are fluctuated.

Many authors (Uysal et al., 2006; Cvijetic et al., 2007.) was researched and analyzed in detail the diverse techniques to mitigate the reliability degradation of FSO systems due to scintillation effects. Uysal et al. (2006) and X. Zhu et al. (2003) presented the performance analysis of coded FSO systems over turbulence channels. T. A. Tsiftsis (2008) studied heterodyne FSO systems over Gamma-Gamma distribution. Also, I. B. Djordjevic et al. (2006) and N. Cvijetic et al. (2007) proposed a MIMO FSO system to effectively improve performance. In our research, we used homodyne systems to effectively overcome turbulence channels because homodyne systems are more sensitive to optical phase and frequency stability (Kazovsky et al., 1996). Although BER performance in optical systems needs to be analyzed using complex equations, Gaussian Q-function approximations can also be used. Thus, we used a Gaussian Q-function for the conditional BER (Humblet et al., 1991; Marcuse, 1991). Also, to provide a more efficient turbulence channel model, we used a Gamma-Gamma distribution (Uysal et al., 2006; Tsiftsis, 2008) which clearly shows scintillation effects. Gamma-Gamma distribution can be directly applied to atmospheric conditions and provides a good fit for experimental results. Consequently, based on homodyne systems and Gamma-Gamma distribution, we analyzed the average BER according to the average SNR.

2. Atmospheric turbulence channel model

Fig. 1 represents the FSO systems over atmospheric turbulence channels. When the optical signals propagate through turbulence channels, they are distorted due to scintillation effects. Gamma-Gamma distribution is represented by the product of small-scale and large-scale irradiance fluctuations and both of these have gamma distribution. Gamma-Gamma distribution is (Uysal et al., 2006; Tsiftsis, 2008).

$$f_I(I) = \frac{2(\alpha\beta)^{\frac{\alpha+\beta}{2}}}{\Gamma(\alpha)\Gamma(\beta)\bar{I}}\left(\frac{I}{\bar{I}}\right)^{\frac{\alpha+\beta}{2}-1} \mathrm{K}_{\alpha-\beta}\left(2\sqrt{\alpha\beta\left(\frac{I}{\bar{I}}\right)}\right) \quad (1)$$

Where, $I > 0$, I is the average irradiance of the channel, α and β are the scintillation parameters, $K_{\epsilon}(\cdot)$ is the modified Bessel function of the second kind of order ε, and $\Gamma(\cdot)$ is the Gamma function.

* Corresponding Author.
Email Address: yjsong@kumoh.ac.kr (Y-J Song)

According to the atmospheric conditions, α and β are defined as (Uysal et al., 2006),

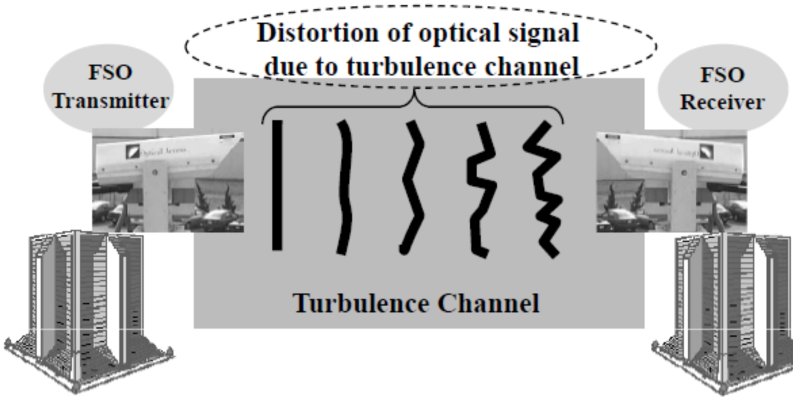


Fig. 1: FSO systems in atmospheric turbulence channels

$$\alpha = \left(exp\left[\frac{0.49\sigma_R^2}{\left(1 + 0.18d^2 + 0.56\sigma_R^{\frac{12}{5}}\right)^{\frac{7}{6}}} \right] - 1 \right)^{-1} \quad (2)$$

$$\beta = \left(exp\left[\frac{0.51\sigma_R^2\left(1 + 0.69\sigma^{\frac{12}{5}}\right)^{-\frac{5}{6}}}{\left(1 + 0.9d^2 + 0.62d^2\sigma_R^{\frac{12}{5}}\right)^{\frac{5}{6}}} \right] - 1 \right)^{-1} \quad (3)$$

where $\sigma_R^2 = 0.5C_n^2\kappa^{7/6}L^{11/6}$ is Rytov variance which has been used as an estimate of the intensity variance, σ_R is the turbulence strength, $d = (\kappa D^2/4L)^{1/2}$, D is the diameter of the receiver collecting lens aperture, L is the link distance in meters, κ is the optical wave number, L is the propagation distance, and C_n^2 stands for the altitude-dependent index of the refractive structure parameter. Several C_n^2 profile models are available in the literature, but the most commonly used is the Hufnagle-Valley model described by

$$\begin{aligned} C_n^2(h) &= 0.005494(v/27)^2(10^{-5}h)^{10}\, exp\left(\frac{h}{100}\right) \\ &+2.7 \times 10^{-6}\, exp\left(-\frac{h}{1500}\right) + Aexp\left(-\frac{h}{1000}\right) \end{aligned} \quad (4)$$

Where h, the altitude in meters (m) is, v is the rms wind-speed in meters per second (m/sec) and A is the nominal value of $C_n^2(0)$ at the ground in $m^{-2/3}$. For FSO links near the ground, C_n^2 can be taken as approximately $1.7 \times 10^{-14}\ m^{-2/3}$ during daytime and 8.4×10^{-15} at night. In general, C_n^2 varies from $10^{-13}m^{-2/3}$ to $10^{-17}m^{-2/3}$.

3. Derivation of the average BER

3.1. Binary signals

We first derive the general form for the average BER of FSO systems with binary signals modulated by On-Off Keying (OOK), Orthogonal coherent BFSK (or BPPM), Antipodal coherent BPSK, Orthogonal non-coherent BFSK, and Antipodal differentially coherent BPSK (or DPSK). In (Djordjevic et al., 2006), the expression of a conditional BER is given as

$$P_b(I) = \frac{\Gamma(b, a\xi_0 I)}{2\Gamma(b)} \quad (5)$$

where $\Gamma(\cdot,\cdot)$ is the complementary incomplete gamma function traditionally defined as $\Gamma(\alpha, x) = \int_x^\infty e^{-t}t^{\alpha-1}d_t$, and a and b depend on the particular form of modulation and detection, as presented in Table 1. Also, $\xi_0 I$ is the signal intensity dependent SNR, where $\xi_0 = \frac{\eta AT}{h\nu}$, η is the quantum efficiency of the detector, A is the detector area, T is the symbol duration, h is Plank's constant, and ν is the frequency of the received signal.

To make a closed-form expression, we introduced the Meijer G function (Adamchik and Marichev, 1990),

$$\mathrm{G}_{\mathrm{p,q}}^{\mathrm{m,n}}\left[\mathrm{x}\middle|{}^{\mathrm{a_1,\cdots,a_p}}_{\mathrm{b_1,\cdots,b_p}}\right] = \frac{1}{2\pi \mathrm{i}}\int \frac{\prod_{\mathrm{j=1}}^{\mathrm{m}}\Gamma(\mathrm{b_j}-\mathrm{s})\prod_{\mathrm{j=1}}^{\mathrm{n}}\Gamma(1-\mathrm{a_j}+\mathrm{s})}{\prod_{\mathrm{j=m+1}}^{\mathrm{q}}\Gamma(1-\mathrm{b_j}+\mathrm{s})\prod_{\mathrm{j=n+1}}^{\mathrm{p}}\Gamma(\mathrm{a_j}-\mathrm{s})}\mathrm{x^s d_s} \quad (6)$$

Where $0 \le m \le q$, $0 \le n \le p$. Also, the Meijer G function supports the following essential equations (Adamchik and Marichev, 1990),

$$erfc(\sqrt{x}) = \frac{1}{\sqrt{\pi}}G_{1,2}^{2,0}\left[x\middle|{}^{1}_{0,\frac{1}{2}}\right] \quad (7)$$

$$K_\nu(x) = \frac{1}{2}G_{0,2}^{2,0}\left[\frac{x^2}{4}\middle|{}^{-}_{\frac{\nu}{2},\frac{-\nu}{2}}\right] \quad (8)$$

Thus, the conditional BER (Table 1) and Gamma-Gamma distribution (Eq. 1) can be represented by the Meijier G function using Eq. 7 and Eq. 8.

The average unconditional BER $\left(\overline{P_b}\right)$can be calculated using the following integral:

$$\overline{P_b} = \int_0^\infty f_I(I)P_b(I)d_I. \quad (9)$$

Then, using the exponential expression and integral of Meijer's G function [6, Eq. 07.34.16.0001.01], a closed-form of the conditional BER is:

$$P_b(I) = \frac{a\xi_0 I}{2\Gamma(b)}G_{1,2}^{2,0}\left[a\xi_0 I\middle|{}^{0}_{-1,-b-1}\right] \quad (10)$$

The average BER is derived as:

$$\overline{P_b} = \frac{a\xi_0}{2\Gamma(\alpha)\Gamma(\beta)\Gamma(b)}\int_0^\infty G_{0,2}^{2,0}\left[\alpha\beta\frac{I}{\bar{I}}\middle|{}^{-}_{\alpha,\beta}\right] G_{1,2}^{2,0}\left[a\xi_0 I\middle|{}^{0}_{-1,-b-1}\right]d_I \quad (11)$$

$$\overline{P_b} = \frac{a\xi_0\bar{I}}{2\Gamma(\alpha)\Gamma(\beta)\Gamma(b)} G_{3,2}^{2,2}\left[\frac{a\xi_0\bar{I}}{\alpha\beta}\Big|\begin{matrix}-\alpha,-\beta,0\\-1,-1-b\end{matrix}\right] \quad (12)$$

Table 1: a and b for various modulations

a \ b	1/2	1
1/4	On-Off Keying (OOK)	
1/2	Orthogonal coherent BFSK (or BPPM)	Orthogonal non-coherent BFSK
1	Antipodal coherent BPSK	Antipodal differentially coherent BPSK (or DPSK)

3.2. Homodyne systems

In this section, we derive the average BER of homodyne systems according to Phase Shift Keying (PSK), Amplitude Shift Keying (ASK), and Frequency Shift Keying (FSK) modulation (Kazovsky et al., 1996). Table 2 shows the conditional BER, $P_b(I)$ of each modulation.

Table 2: Conditional BER, $P_b(I)$ where $Q(x)=1/2erfc(x/\sqrt{2})$

Modulation	Conditional BER
PSK	$Q(2\sqrt{\xi_0 I})$
ASK	$Q(\sqrt{2\xi_0 I})$
FSK	$Q(\sqrt{\xi_0 I})$

By substituting Eq. 1 and the conditional BER with Eq. 9, we can obtain Eq. 13.

$$\overline{P_b} = \frac{(\alpha\beta)^{\frac{\alpha+\beta}{2}}}{2\sqrt{\pi}\Gamma(\alpha)\Gamma(\beta)\bar{I}}\int_0^\infty \left(\frac{I}{\bar{I}}\right)^{\frac{\alpha+\beta}{2}-1} G_{0,2}^{2,0}\left[\alpha\beta\frac{I}{\bar{I}}\Big|\frac{\alpha-\beta}{2},\frac{\beta-\alpha}{2}\right] G_{1,2}^{2,0}\left[2\xi_0 I\Big|\begin{matrix}1\\0,\frac{1}{2}\end{matrix}\right] d_I \quad (13)$$

Subsequently, through transformations and argument simplifications of the Meijer G function [Wolfram, 2004, Eq. 07.34.16.0001.01], Eq. 13 is reduced to Eq. 14.

$$\overline{P_b} = \frac{\alpha\beta}{2\sqrt{\pi}\Gamma(\alpha)\Gamma(\beta)\bar{I}}\int_0^\infty G_{0,2}^{2,0}\left[\alpha\beta\frac{I}{\bar{I}}\Big|1-\alpha,1-\beta\right] G_{1,2}^{2,0}\left[2\xi_0 I\Big|\begin{matrix}1\\0,\frac{1}{2}\end{matrix}\right] d_I \quad (14)$$

$$\int_0^\infty x^{\alpha-1} G_{u,v}^{s,t}\left[\sigma x\Big|\begin{matrix}c_1,\cdots,c_t,c_{t+1},\cdots,c_u\\d_1,\cdots,d_s,d_{s+1},\cdots,d_u\end{matrix}\right] G_{p,q}^{m,n}\left[\omega x\Big|\begin{matrix}a_1,\cdots,a_n,a_{n+1},\cdots,a_p\\c_1,\cdots,c_t,c_{t+1},\cdots,c_u\end{matrix}\right] d_x$$

Finally, using classical Meijer's integral (Eq.15) of the two G functions,

$$= \frac{k^\mu l^{\rho+\alpha(v-u)-1}\sigma^{-\alpha}}{(2\pi)^{b^*(l-1)+c^*(k+1)}} G_{kp+lv,kq+lu}^{km+lt,kn+ls}\left[\frac{\omega^k k^{k(p-q)}}{\sigma^l l^{l(u-v)}}\Big|\begin{matrix}\Delta(k,a_1),\cdots,\Delta(k,a_n),\Delta(l,1-\alpha-d_1),\cdots,\Delta(l,1-\alpha-d_v),\Delta(k,a_{n+1}),\cdots,\Delta(k,a_p)\\\Delta(k,b_1),\cdots,\Delta(k,b_m),\Delta(l,1-\alpha-c_1),\cdots,\Delta(l,1-\alpha-c_u),\Delta(k,b_{m+1}),\cdots,\Delta(k,b_q)\end{matrix}\right] \quad (15)$$

where $\Delta(k,a) = \frac{a}{k},\frac{a+1}{k},\cdots,\frac{a+k-1}{k}$

$b^* = s+t-(u+v)/2, \rho = \Sigma_{j=1}^{v} d_j - \Sigma_{j=1}^{u} c_j + (u-v)/2+1,$

$c^* = m+n-(p+q)/2, \mu = \Sigma_{j=1}^{q} b_j - \Sigma_{j=1}^{p} a_j + (p-q)/2+1$

A closed-form expression for the average unconditional BER of PSK scheme is written as:

$$\overline{P_b} = \frac{1}{2\sqrt{\pi}\Gamma(\alpha)\Gamma(\beta)} G_{3,2}^{2,2}\left[\frac{2\xi_0\bar{I}}{\alpha\beta}\Big|\begin{matrix}1-\alpha,1-\beta,1\\0,\frac{1}{2}\end{matrix}\right] \quad (16)$$

4. Numerical results

Analysis results for a closed-form expression of the average BER versus the average SNR are shown in Fig. 2.

As revealed by the numerical results, under different turbulence strength assumptions, the average BER for the antipodal coherent BPSK provides the best performance, with OOK having the least effective performance for every turbulence strength.

Fig. 3 then represents the average BER according to beta, where SNR is 25 dB. In this case, when the turbulence channel strength becomes weak (β increases), the average BER of the orthogonal no coherent BFSK is similar to the orthogonal coherent BFSK, and the average BER of the antipodal differentially coherent BPSK is similar to the antipodal coherent BPSK.

The results of homodyne FSO systems for a closed-form expression are displayed in Fig. 4 (a), (b), and (c) illustrating PSK, ASK, and FSK, respectively. Then, as an example, we considered different turbulence strengths: $(\alpha,\beta) \in \{(4,1),(4,2),(4,4)\}$ and compared them with no turbulence channels.

Fig. 4 represents the comparison of the average BER performance including PSK, ASK, and FSK as a function of average SNR according to different turbulence strengths. As β decreases, the turbulence effects become stronger. It is observed that these plots cover relatively weak turbulence where the average BER performance decays very quickly when the average SNR increases. As revealed by the numerical results, under different turbulence strength assumptions, the average BER for PSK provides the best performance, and FSK gives the least effective performance in every turbulence strength. These results are the same the adaptive white Gaussian noise (AWGN) channels.

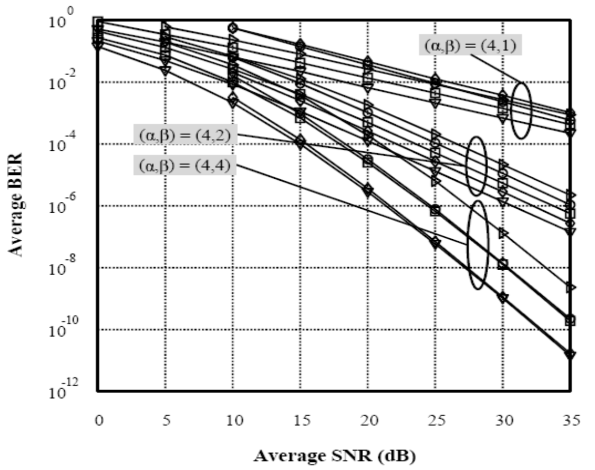


Fig. 2: Average BER as a function of average SNR

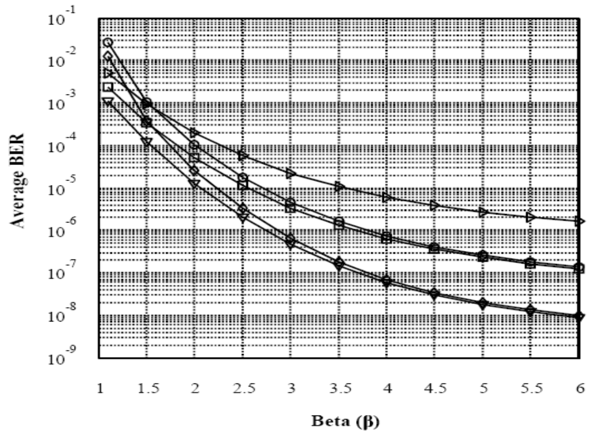
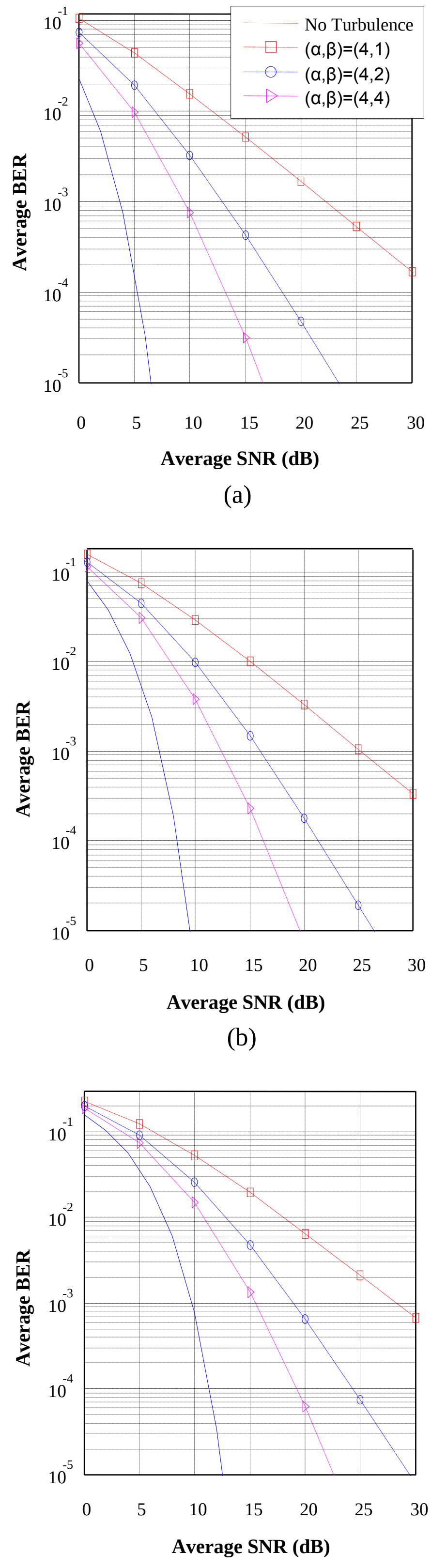


Fig. 3: Average BER as a function of β under SNR = 25 dB

5. Conclusions

In this paper, we present a closed-form for the average BER of FSO system using special integrals and transformations of the Meijer G function. Additionally, the average BER performance behavior of FSO systems over atmospheric turbulence channels with Gamma-Gamma distribution is analyzed. Based on a closed-form expression for the average BER, a generalized approach is adopted to consider PSK, ASK, and FSK schemes with various turbulence strengths. The results are further illustrated to confirm the analysis. As a result of our research, we are able to provide a simpler and more reliable method of estimating BER performance, thus removing the need for more complex calculations. Practically, when we establish FSO systems, we can make the engineering table using this derived BER equations according to each modulation scheme. Also, we can choose the efficient modulation scheme in accordance with the environment.



(c)

Fig. 4: The average BER as a function of photons per bit for (a) PSK (b) ASK (c) FSK according to no turbulence channels and in $(\alpha,\beta) \in \{(4,1),(4,2),(4,4)\}$.

Acknowledgment

This research was supported by the MSIP (Ministry of Science, ICT and Future Planning), Korea, under the "ICT Consilience Creative Program" (IITP-2015-R0346-15-1007) supervised by the IITP (Institute for Information and communications Technology Promotion)

References

Adamchik VS and Marichev OI (1990). The algorithm for calculating integrals of hypergeometric type functions and its realization in REDUCE system. In Proceedings of the international symposium on Symbolic and algebraic computation (pp. 212-224). ACM.

Cvijetic N, Wilson SG and Brandt-Pearce M (2007). Receiver optimization in turbulent free-space optical MIMO channels with APDs and Q-ary PPM. Photonics Technology Letters, IEEE, 19(2): 103-105.

Djordjevic IB, Vasic B and Neifeld MA (2006). Multilevel coding in free-space optical MIMO transmission with Q-ary PPM over the atmospheric turbulence channel. IEEE photonics technology letters, 18(13/16): 1491.

Humblet PA and Azizoğlu M (1991). On the bit error rate of lightwave systems with optical amplifiers. Journal of Lightwave Technology, 9(11): 1576-1582.

Kazovsky LG, Benedetto S and Willner AE (1996). Optical fiber communication systems. Artech House, Norwood, USA.

Lim W (2015). BER Analysis of Coherent Free Space Optical Systems with BPSK over Gamma-Gamma Channels. Journal of the Optical Society of Korea, 19(3): 237-240.

Marcuse D (1990). Derivation of analytical expressions for the bit-error probability in lightwave systems with optical amplifiers. Journal of Lightwave Technology, 8(12): 1816-1823.

Tsiftsis TA (2008). Performance of heterodyne wireless optical communication systems over gamma-gamma atmospheric turbulence channels. Electronics Letters, 44(5): 373-375.

Uysal M, Li J and Yu M (2006). Error rate performance analysis of coded free-space optical links over gamma-gamma atmospheric turbulence channels. Wireless Communications, IEEE Transactions on, 5(6): 1229-1233.

Wolfram (2004) The Wolfram function site. Available: http://functions.wolfram.com.

Zhu X and Kahn JM (2003). Performance bounds for coded free-space optical communications through atmospheric turbulence channels. Communications, IEEE Transactions on, 51(8): 1233-1239.